COMPOSITIONALITY AND STRUCTURED PROPOSITIONS

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ABSTRACT

In this paper, we evaluate an influential argument for Structured Propositionalism: the widely held theory according to which sentential semantic values are structured, abstract entities with constituents. The argument hinges on two seemingly innocuous and widely accepted premises: the Principle of Semantic Compositionality and Propositionalism (the thesis that sentential semantic values are propositions). We object to the way that compositionality has to be construed in order to render the argument sound. Specifically, we argue that the Compositionality Argument, as we call it, presupposes that compositionality involves a form of building, and that this metaphysically robust account of compositionality is subject to counter-example: there are compositional representational systems that this principle cannot accommodate. If this is correct, one of the most important arguments supporting Structured Propositionalism is undermined.

§0. Introduction

Structured Propositionalism—the thesis that propositions have as constituents or parts the semantic values of the meaningful parts of the sentences that express them—has the status of near-orthodoxy among philosophers of language. Yet arguments for this thesis are scarce. One exception is the “negative” argument that Structured Propositionalism is superior to the Possible Worlds View, according to which propositions are sets of worlds. However, an argument for the superiority of one view to another is only successful if there are no alternatives. And there are alternatives—for example, the Primitive Entity Theory, according to which propositions are sui generis, simple, abstract entities with fine-grained truth-conditions. So Structured Propositionalism’s superiority to the Possible Worlds View is insufficient to establish Structured Propositionalism.

1 Thanks to Mahrad Almotahari, Julien Dutant, Daniel Harris, Curtis Kehler, Jeff Speaks, and Paula Sweeney for helpful comments and discussion.

2 See e.g. Jeff King: “most [propositionalists] assume that an account of propositions on which they are structured entities with constituents and so are individuated more finely than sets of worlds is desirable” (2007: 3). Other structured propositionalists include Scott Soames, Nathan Salmon, David Braun, Kent Bach, and Jason Stanley. Even some advocates of the Possible Worlds View countenance structured intensions: see David Chalmers 2006 and Robert Stalnaker 2012. Early analytics like Gottlob Frege, Bertrand Russell, Bernard Bolzano, and G. E. Moore were also Structured Propositionalists.

3 The locus classicus for this argument is Soames 1987.

4 On this view, propositions are theoretical primitives that are defined by their roles in semantics, psychology, metaphysics, and logic. George Bealer is an influential defender of the Primitive Entity Theory. See, e.g., Bealer 1998; see also see Jubien 2001, Merricks 2009, Plantinga 1974, and van Inwagen 2004. The Primitive Entity Theory (or something very similar) is also discussed in Lewis 1986 and van Inwagen 1986.

5 Especially given the metaphysical puzzles that face Structured Propositionalism. See Keller forthcoming.
Why are positive arguments for Structured Propositionalism so rare? We submit that this is so largely because the arguments for Structured Propositionalism seem obvious in light of the roles propositions are posited to play—most importantly for our purposes, the role of sentential semantic values. However, because these “obvious” arguments are assumed, rather than carefully presented, they are not subjected to scrutiny. As a result, Structured Propositionalism enjoys an unwarranted veneer of unassailability.

One important such “obvious” argument is the argument from semantic compositionality. As Stephen Schiffer says: “Virtually every propositionalist accepts [semantic compositionality] and rejects unstructured propositions” (2003: 18). In a nutshell, what we call the ‘Compositionality Argument’ contends that compositionality entails complexity: if the assignment of semantic values to sentences obeys the principle of compositionality, then sentential semantic values must be complex.

Frege presents the argument thus:

It is astonishing what language can do. With a few syllables, it can express an incalculable number of thoughts, so that even if a thought has been grasped by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would not be possible if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought.6

Russell also gives a version of the argument:

[T]he phrase ‘Roses are red’ can be understood if you know what ‘red’ is and what ‘roses’ are, without ever having heard the phrase before. That is a clear mark of what is complex. It is the mark of a complex symbol, and also the mark of the object symbolized by the complex symbol. That is to say, propositions are complex symbols, and the facts they stand for are complex.7

The Compositionality Argument, along with the argument from logical form, is one of the two sturdiest pillars supporting Structured Propositionalism.8 In what follows, we show that this pillar is not as sturdy as Structured Propositionalists may think.

The paper proceeds as follows: in §1, we provide a more thorough definition of Structured Propositionalism; in §2, we present the Compositionality Argument; in §3, we present an objection; and in §4, we consider Structured Propositionalist responses to the objection. We show that, while Fregeans may have a reply to the objection (although we

6 Italics added. Frege 1923/1984: 390. This argument also appears in Frege 1914/1979.
7 Russell 1918/1956: 195 (emphasis added). Confusingly, Russell uses the word ‘proposition’ in these lectures to mean ‘sentence in the indicative’ (185). Roughly, Russell’s structured facts are what we would call propositions. The relation between Russell’s ‘propositions’ and facts is complex: “the components of the fact which makes a proposition true or false, as the case may be, are the meanings of the symbols which we must understand in order to understand the proposition” (196). More needs to be said, but there is insufficient space here for thorough Russell exegesis.
8 The Logical Form Argument is discussed in Keller 2012. Nathan Salmon presents an enthymematic version of the Compositionality Argument in Salmon 1989. It is not uncommon for introductory philosophy of language textbooks to slide from the principle of compositionality to the thesis that propositions are structured. See, e.g., Kemp 2013: 7-8.
doubt it), Russellians do not. Thus our objection undercuts one of the central arguments for the most popular contemporary version of Structured Propositionalism.

§1. Structured Propositionalism Defined

Structured Propositionalism can be defined as the view that a proposition $p$ expressed by a sentence $s$ in a context $c$ is a complex entity that is composed of the semantic values of the meaningful parts of $s$ in $c$, and has a structure that in some way mirrors that of $s$ (or the logical form, or LF, of $s$).\(^9\) Thus on Structured Propositionalism, propositions have constituents, and they are individuated by the identity and arrangement of their constituents.\(^10\) Beyond this general statement, there is disagreement about how to flesh out the view: specifically, about how propositions are structured, about how propositional structure is related to sentence structure, and about the nature of propositional constituents.

A theory of the nature of propositional constituents is provided by the attendant semantics. According to Russellians, a sentence such as ‘Elvis sings’ expresses a complex entity composed of the contents of ‘Elvis’ and ‘sings’—i.e., Elvis himself and the property of singing. According to Fregeans, Elvis and the property of singing are replaced by a mode-of-presentation of Elvis and the concept of singing (or the senses expressed by ‘Elvis’ and ‘sings’).\(^11\)

We will use ‘semantic value’ as a neutral term for whatever type of meaning makes a propositional contribution (Russellian content or Fregean sense). On both views, logically complex propositions are “built up” from atomic propositions (or propositional functions) by application of recursive rules, and have as constituents the semantic values of truth-functional connectives, quantifier expressions, etc. Distinct propositions with identical constituents, such as John loves Maggie and Maggie loves John, are distinguished in terms of their structure.

When it comes to paradigmatic singular terms such as names, demonstratives, and variables, most Russellians hold that content and reference coincide, and so the semantic values of such terms are their referents. Although contemporary Russellians typically invoke Kaplan’s character-content distinction in their semantics, since the Compositionality Argument is an argument for structured propositions, it is content that is relevant. For the proposition expressed by a sentence $s$ is determined by the contents of $s$’s constituent expressions.\(^12\) By contrast, Fregeans countenance both a sense and a reference (if all goes well) of any significant term. For Fregeans, however, the sort of semantic value relevant to the Compositionality Argument is sense, since they hold that the proposition expressed by a sentence $s$ is determined by the senses of $s$’s meaningful parts.

§2. The Compositionality Argument

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\(^9\) We will drop the reference to context below, as well as the niceties about logical form or LF. For the view that propositional structure mirrors sentential structure at LF, see King 2007. For a similar view invoking phrase structure markers, see Richard 1990.


\(^11\) There are other positions that combine Fregean and Russellian approaches—what Schiffer calls ‘Fresselian’ theories—but considerations of space and simplicity counsel against discussing them here.

\(^12\) Some contemporary Russellians eschew sentence-level characters (see King and Stanley 2005), and so obviously cannot invoke character in an account of the nature of propositions. But even for those who do invoke sentence-level characters, they play the role of guises, not propositions. See, e.g., Braun 2006.
The following is a widely accepted statement of the principle of compositionality:

(C) The semantic value of a complex expression is determined by the semantic values of its parts and the way that they are arranged (i.e., its structure).

This formulation has the virtue of being amenable to wide endorsement; however, this is because it has the vice of being terribly vague. What is meant by 'semantic value'? Which parts of a complex expression are the relevant ones for determining its semantic value? And what is meant by ‘determined’ anyway?

The answer to the first of these questions will depend on the semantic theory at issue, which we will discuss at the end of the paper. The second is a question of what the ultimate syntactic categories are—an issue better left to linguists. But the answer to the third question is of direct relevance to our evaluation of the Compositionality Argument. Assuming we have a working semantics and a syntax that tells us what parts of expressions are relevant to determining the semantic values of complex expressions, how does such determination take place? What does it involve?

The Structured Propositionalist has a simple and somewhat attractive answer: the semantic values of the parts $e_1\ldots e_n$ of an expression $e$ are parts of the semantic value of $e$. Thus complex expressions have complex semantic values. We will call the resulting construal of compositionality the ‘Building Principle’:

(B) The semantic value of a complex expression $e$ is (i) built from, or composed out of, the semantic values of $e$’s parts, and (ii) has a structure corresponding to the syntactic structure of $e$.

Quite clearly, if (B) is the correct understanding of compositionality, then we have a straightforward argument for Structured Propositionalism:

The Compositionality Argument:

(1) The semantic value of a complex expression $e$ is composed out of the semantic values of $e$’s parts and has a structure corresponding to the syntactic structure of $e$.

(2) Propositions are the semantic values of certain complex expressions, viz. (declarative, univocal) sentences.

(3) Hence, propositions are structured entities composed out of the semantic values of the parts of the sentences that express them.

This argument is clearly valid. Since (2) is a platitude almost all propositionalists accept, a propositionalist looking to avoid the conclusion has one way to go: attack (1).

In order to assess (1) we have to ask whether the only or best way to make sense of compositionality involves building. If not, then acceptance of the highly plausible claim that natural language is compositional will not require acceptance of (B). One straightforward test for candidate formulations of the principle of compositionality is to see whether they can do

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the work that compositionality is invoked to perform. Since the reasons for accepting compositionality are well known, we will just briefly outline them here.

Friends of compositionality typically point to the productivity, understandability, and systematicity of language, and argue that compositionality is the best, or only, way to explain these features. A representational system \( L \) is **productive** just in case finite beings can use \( L \) to produce an infinite number of meaningful complex expressions; it is **understandable** just in case someone competent with \( L \) (i.e., who grasps the finite lexicon and grammatical rules of \( L \)) is capable of understanding complex expressions of \( L \) she has never before encountered; and it is **systematic** just in case whenever someone competent with \( L \) is capable of understanding an expression \( e \) of \( L \), she is capable of understanding systematic variants of \( e \) (expressions obtained by permuting the constituents of \( e \)). For example, anyone who understands expressions of the form \( Fa \) and \( Gb \) can also understand expressions of the form \( Ga \) and \( Fb \).\(^{14}\)

The principle of compositionality is supposed to explain these features. Assuming that (C) lives up to this explanatory role, it follows that (B) can play the role as well, since (B) entails (C)—if the SVs of the complex expressions of a language \( L \) are built from the SVs of those expressions’ parts, etc., then it follows that the SVs of complex expressions in \( L \) are determined by the semantic values of their parts plus syntax. Hence, if (C) accounts for these features, so can (B).

Whether (B) actually does explain the productivity, understandability, and systematicity of a given representational system, however, depends on whether (B) is in fact satisfied by that system. In what follows, we argue that there are compositional systems that do not satisfy (B), but which obey a weaker principle that is nonetheless capable of accounting for the compositional features of language more generally.

§3. The Function Objection

The problem with (B) is that satisfying it is not necessary for compositionality, since there are compositional representational systems that violate it. These systems are productive, understandable, and systematic, and they satisfy both (C) and a weaker principle that is sometimes called the ‘Function Principle’:

\[(F) \quad \text{The semantic value of a complex expression is a function of the semantic values of that expression’s parts and their arrangement.}^{15}\]

The Arabic and Roman numeral systems are both compositional systems that satisfy (F), but not (B). The numerals are systems for representing an infinite number of entities with finite means. In both systems, complex expressions are built up recursively from a finite stock of primitive expressions. The semantic values of the numerals are, presumably, the numbers

\[^{14}\text{Systematicity is harder to define than productivity and understandability, and there is more (although still little) controversy over whether language is systematic than whether it is productive and understandable. Our definition of systematicity is taken from Cummins 1996. See Johnson 2004 for discussion.}\]

\[^{15}\text{Cf. Harnish 2008, Fodor and Pylyshyn 1989, Pelletier 1994, and Szabö 2000. Also, see Fodor and LePore 2002 for a broad account of compositionality that applies not just to language but to representational systems in general.}\]
they designate; i.e., the numeral ‘2’ refers to, and has as its semantic value, the number two. But if this is so, then the numeral systems cannot be governed by (B).\footnote{What we say here makes widely accepted, but still controversial, assumptions about the nature of numbers and number discourse. (See Steiner 2011 for an interesting critique of orthodoxy on these matters.) These assumptions are dispensable: the Cartesian and polar coordinate systems provide the means for an argument that closely parallels the one given here.}

Consider the numeral ‘19’: it is composed of the expressions ‘1’ and ‘9’, combined recursively (‘1’ is in the $10^1$ place, ‘9’ in the $10^0$ place) to form an expression that designates the number nineteen. The semantic values of the expressions that compose ‘19’ are the numbers one and nine. When those numbers are plugged into the equation $n \times 10^1 + m \times 10^0$ the result is the number nineteen. But is there any sense in which the numbers one and nine are parts of the number nineteen? This claim is highly implausible on its own, and it doesn’t follow from the compositional nature of the Arabic numeral system. For consider the co-referring Roman numeral, ‘XIX’, which is built from the simple expressions ‘X’ and ‘I’, designating ten and one (following a different recursive pattern than the Arabic system). If the Building Principle were the correct theory of compositionality, it would follow that the number denoted by ‘XIX’ is composed of the numbers ten and one, while the number denoted by ‘19’ is composed of the numbers one and nine. But ‘XIX’ and ‘19’ co-refer, so the Building Principle entails that the number nineteen both has, and lacks, ten as a part. This is an obvious contradiction.

One might think that this result can be avoided by claiming that ‘19’ is composed of ‘10’ and ‘9’, rather than ‘1’ and ‘9’. If we also claim that ‘XIX’ is composed of ‘X’ and ‘IX’, then nineteen’s decomposition can mirror that of both ‘19’ and ‘XIX’.\footnote{Thanks to Paula Sweeney for raising this objection.}

This move seems \textit{ad hoc} to us, however, and doesn’t help with the fact that there are simple Roman numeral expressions (such as ‘X’) that are co-referential with complex Arabic expressions (such as ‘10’), and vice versa (‘VI’ and ‘6’). The Building Principle entails that the semantic value of ‘10’, but not ‘X’, has one and zero as parts; \textit{mutatis mutandis} for ‘6’ and ‘VI’. This is impossible: simple numerals cannot refer only to simple numbers while complex numerals refer only to complex numbers. But if this cannot be, then it cannot be required by compositionality, since both the Arabic and Roman numeral systems are clearly compositional. Both systems are productive, understandable, and systematic: finite beings can use the systems to represent an infinite domain of numbers; we can understand complex expressions such as ‘2,196,354,248’ that we have never before encountered; and anyone who understands, e.g., ‘57’ will also understand ‘75’, and vice versa (similarly for ‘VI’ and ‘IV’).

The upshot is that at most one of the Arabic and Roman Numeral systems can satisfy (B). Since both are productive, understandable, and systematic, it follows that satisfying (B) is not necessary for a representational system to have these properties, i.e., to be compositional.\footnote{One might object that the complexity of the complex numerals is \textit{morphological} rather than \textit{syntactic}, and so a different sort of “compositionality” would apply in the case of the numeral systems. It is unclear to us that there are different kinds of compositionality, but even if morphological and syntactic compositionality are distinct, our argument shows that morphological compositionality does not require complexity, since the numeral systems satisfy do not satisfy (B). A defender of the Compositionality Argument would then need to argue that syntactic compositionality requires a stronger principle than the one accounting for morphological compositionality. It is hard to see how such an argument would run.}
So the numeral systems are compositional, but their compositional properties are not explained by (B). The Function Principle, however, nicely explains their compositional properties. For example, if you plug in the semantic values of ‘1’ and ‘9’ into the relevant slots of the function \( _1 \times 10^1 + _0 \times 10^0 \), the result is nineteen, the semantic value of ‘19’. Hence the semantic value of the complex expression ‘19’ is a function of the semantic values of the primitive expressions from which ‘19’ is composed and the way that they are arranged. In general, for a complex numeral ‘\(^n_3 \times 10^3 + n_2 \times 10^2 + n_1 \times 10^1 + n_0 \times 10^0\)’ in the Arabic system, the function \(n_3 \times 10^3 + n_2 \times 10^2 + n_1 \times 10^1 + n_0 \times 10^0\) will yield its semantic value. It is easy to see how this function can be expanded recursively to yield numbers as semantic values for numerals of increasing length.

So the Arabic and Roman numeral systems are compositional, but they do not satisfy (B). They do, however, satisfy (F). It seems, then, as if (F) is a better way of rendering compositionality than (B). Since the Arabic and Roman numeral systems are productive, understandable, and systematic, and obey (F) but not (B), systems need not satisfy (B) in order to be compositional—(F) will suffice.

§4. Responses to the Function Objection

A Fregean might respond to the Function Objection as follows: though the references of the numerals are the numbers, the senses of the numerals are complex entities determining the numbers to which they refer. It is the senses of numerals rather than their referents that are constituents of the relevant propositions.\(^{20}\) Further, there is nothing implausible about saying that the senses of the numeral expressions obey (B) and that the semantic values of complex numerals are therefore complex. For Fregeans, this in no way implies that the numbers themselves have parts corresponding to the parts of the numerals that designate them. In addition, it is no problem that Roman and Arabic numerals designating the same number may express different senses, since the relation of sense to reference is many-one. Thus, at least at first glance, it appears that Fregeans are able to circumvent the Function Objection.

If the Building Principle is correct, however, ‘VI’ and ‘6’ must have different senses, the former composed out of the senses of ‘V’ and ‘I’, the latter being simple. And it is far from clear that that ‘VI’ and ‘6’ do have different senses, any more than ‘V’ and ‘5’ or ‘fünf’ and ‘five’ do. Still, we think we can afford to let Fregeans off the hook for now, since most Structured Propositionalists are Russelians.\(^{21}\)

Russellians face a more serious problem, since they apparently must hold that the numerals are complex referring expressions but cannot hold, on pain of contradiction, that their semantic values are built out of the semantic values of the expressions that compose them; i.e., they cannot hold that they are compositional in the sense of (B). Moreover, Russellians cannot adopt the Fregean strategy, since they disavow senses in their semantics.

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\(^{19}\) (F) will almost certainly need to be restricted in some way, since mathematical functions are cheap and there will be mappings of simple expressions-plus-syntax onto complex expressions for languages that are not plausibly compositional. For a discussion of possible restrictions, see Szabó 2000.

\(^{20}\) It is worth pointing out that Frege himself countenanced two compositionality principles, one for sense (very much like (B) above) and one for reference (very much like (F) above).

\(^{21}\) Thanks to Mahrad Almotahari and Julien Dutant for helpful discussion on this point.
It may seem as if there is a way out for Russelians: they can say that the numeral expressions are not names for the numbers, but definite descriptions. If they handle definite descriptions in the manner of the Russell of “On Denoting,” then they can deny that the expressions composing the complex numerals contribute the numbers they would denote in isolation to the semantic values of the complex expressions they compose (e.g., they can deny that ‘1’ contributes the number one to the semantic value of ‘19’). Rather, they will hold that the complex numerals have descriptive contents. So, where ‘F’ is a complex predicate that nineteen uniquely satisfies, the semantic value of ‘19’ would have the form:

$$\exists x (Fx & \forall y (Fy \rightarrow (x = y)))$$

Hence, if Russelians treat numerals (at least the complex numerals) as descriptions, they may be able to avoid the implausible consequences that result from the conjunction of (B) and the view that the numerals are number-names.

In order to assess this reply, we must distinguish views according to which numerals are disguised descriptions from views according to which they merely have descriptive content. The former is unattractive, since most contemporary Russelians are also Kripkeans, and so would not want to follow Russell in treating expressions that are syntactic names as disguised descriptions. However, only the simple numerals are syntactically simple: it is a presupposition of the Function Objection that the complex numerals are syntactically complex, since they exhibit compositional structure. So perhaps some Russelseans will be comfortable denying that complex numerals are names.

However, although the numerals are syntactically complex, they share no syntactic features with descriptions, as they contain no determiners that would function as quantifiers in their logical form. For example, the numeral ‘19’ consists solely of the simple numerals ‘1’ and ‘9’, which are certainly not descriptions. What could they be other than names for the numbers 1 and 9? Furthermore, as noted above, the semantic value of ‘19’ is plausibly determined by the function \(n_1 \times 10^1 + n_0 \times 10^0\)—i.e., nineteen. So treating the numerals as descriptions (and their semantic values as description-like) seems like a desperate move. Among other unpalatable consequences, this would involve interpreting sentences like ‘10 > 9’ as expressing complex quantified claims, rather than singular propositions about numbers. In general, to treat expressions that are syntactically name-like and that share none of the relevant syntactic features of descriptions as disguised descriptions is to reject Kripke’s revolution, of which most contemporary Russelians are ardent supporters.

If Russelians merely hold that numerals have descriptive content, however, their theory will still violate the Building Principle, and so invalidate the Compositionality Argument. If the semantic value of ‘19’ has the form

$$\exists x (Fx & \forall y (Fy \rightarrow (x = y)))$$

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22 Russell himself treated the numerals as disguised descriptions of classes of classes, but Russellian logicism is currently not in vogue. See Russell 1903 and 1910-1913.

23 It’s worth noting that ordinary English names may sometimes take determiners, and in some languages this is quite common. But this complexity does not suggest that names are actually disguised descriptions. Of course, some expressions that lack one of the characteristic features of descriptions (e.g., determiners) may function syntactically like descriptions; for instance, some philosophers hold that expressions like ‘my sister’ and some anaphoric pronouns are best treated as descriptions (see Ludlow 1994, 2007). Our point is that having the relevant syntax is neither necessary nor sufficient for being a (Russelian) description.

then it clearly does not have a structure corresponding to the structure of ‘19’, nor is it composed of the semantic values of ‘1’ and ‘9’. Note that, on our view, the semantic value of ‘19’ is determined by the function \((n_1 \times 10^1 + n_0 \times 10^0)\), not identical to it—we hold that the semantic value of ‘19’ is nineteen. Suppose that the semantic value of ‘19’ were the function \((1 \times 10^1 + 9 \times 10^0)\), and that this function was not a set of ordered pairs but something structurally isomorphic to ‘\((1 \times 10^1 + 9 \times 10^0)\)’. Then, while its structure would arguably correspond to that of ‘19’, it would have constituents that violate the Building Principle, since it would have constituents other than the semantic values of ‘1’ and ‘9’.

We conclude that, while Fregeans may have a potential line of response to the Function Objection, it constitutes a substantial challenge to Russelians. Given the importance of the Compositionality Argument, Russelians ought to be concerned about this challenge. And given the prominence of Russellianism among proponents of structured propositions, the community of Structured Propositionalists cannot afford to ignore the challenge either.

REFERENCES


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